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Title: The Problem with Codomains

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Abstract

A function f typically consists of three parts:

- a set called the *domain* of f, which we denote by $\operatorname{dom}(f)$;
- a set called the *codomain* of f, which we denote by $\operatorname{cod}(f)$;
- a *mapping rule*, which is some expression that, given $x \in \operatorname{dom}(f)$, names an element of $\operatorname{cod}(f)$, called the *image* of x under f and denoted by $f(x)$; the element x is called the *argument* in this context, and we say that x is *mapped* to $f(x)$ or that f *assumes* the value $f(x)$ in x.

Define:

$$\operatorname{img}(f) := \{f(x) \mid x \in \operatorname{dom}(f)\} \subseteq \operatorname{cod}(f) \qquad \textit{image} \text{ of function } f$$

So the image of f is the set of all values that f assumes.

In this article, I argue that a function should instead only consist of a domain and a mapping rule, no codomain. I will show that:

- in most situations, the codomain is irrelevant;
- when it is not irrelevant, we can look at the image instead;
- if taken seriously, the codomain can make things extremely complicated and evoke serious problems, without providing any benefit.

1 Preliminaries

A common notation is:

$$f\colon A \longrightarrow B :\Longleftrightarrow\ f \text{ is a function} \wedge \operatorname{dom}(f) = A \wedge \operatorname{cod}(f) = B$$

A common notation to define a function is:

$$f\colon A \longrightarrow B,\ x \mapsto E$$

This defines a function f with domain A, codomain B, and for all $a \in A$, we obtain $f(a)$ by replacing each free occurrence of the symbol x by a in the expression E. Of course, any symbol that does not clash with the context can be used instead of x. A variant of this notation can be used to describe an anonymous function:

$$A \longrightarrow B,\ x \mapsto E$$

Equality of functions is defined as:

$$f = g :\Longleftrightarrow\ \operatorname{dom}(f) = \operatorname{dom}(g) \wedge \operatorname{cod}(f) = \operatorname{cod}(g) \wedge (\forall x \in \operatorname{dom}(f) :\ f(x) = g(x))$$

Examples:

- $f\colon \mathbb{R} \longrightarrow \mathbb{R},\ x \mapsto x^2$ is the function that maps each real to its square and which has codomain $\mathbb{R}$. The function $g\colon \mathbb{R} \longrightarrow \mathbb{R}_{\geq 0},\ x \mapsto x^2$ maps each real to its square and has codomain $\mathbb{R}_{\geq 0}$, hence $f \neq g$.
- $f\colon \mathbb{R} \longrightarrow \mathbb{R}_{>0},\ x \mapsto x^2$ is not a function, because for argument 0, the mapping rule produces 0, which is not an element of $\mathbb{R}_{>0}$.
- $f\colon \mathbb{R} \longrightarrow \mathbb{R},\ x \mapsto \frac{1}{x}$ is not a function, because the mapping rule cannot be applied to argument 0.
- $f\colon \mathbb{R} \longrightarrow \mathbb{R},\ x \mapsto (x+1)^2$ is a function, and the same function as $g\colon \mathbb{R} \longrightarrow \mathbb{R},\ x \mapsto x^2 + 2x + 1$ as per our equality definition.

Define:

$$f \text{ is } \textit{injective} :\Longleftrightarrow\ \forall a, b \in \operatorname{dom}(f) :\ f(a) = f(b) \implies a = b$$
$$f \text{ is } \textit{surjective} :\Longleftrightarrow\ \operatorname{img}(f) = \operatorname{cod}(f)$$
$$f \text{ is } \textit{bijective} :\Longleftrightarrow\ f \text{ is injective} \wedge f \text{ is surjective}$$
$$g\colon \operatorname{cod}(f) \longrightarrow \operatorname{dom}(f) \text{ is an } \textit{inverse} \text{ of } f :\Longleftrightarrow\ g \circ f = \mathrm{id}_{\operatorname{dom}(f)} \wedge f \circ g = \mathrm{id}_{\operatorname{cod}(f)}$$

Remarks:

- The notion of injectivity does not use the codomain; we can decide whether a function is injective without ever looking at its codomain.
- f is bijective $\iff$ f admits an inverse
- When f admits an inverse, then this inverse is uniquely determined and denoted by f^{-1}.

Define:

$$f|_M\colon M \longrightarrow \operatorname{cod}(f),\ x \mapsto f(x) \qquad \textit{restriction} \text{ of function } f \text{ to set } M \subseteq \operatorname{dom}(f)$$

Examples:

- $f\colon \mathbb{R}_{\geq 0} \longrightarrow \mathbb{R},\ x \mapsto x^2$ is injective but not surjective. The function $g\colon \mathbb{R}_{\geq 0} \longrightarrow \mathbb{R}_{\geq 0},\ x \mapsto x^2$, which differs from f only by its codomain, is bijective, and g^{-1} is denoted by $\sqrt{\cdot}$.
- The *real exponential function* $\exp\colon \mathbb{R} \longrightarrow \mathbb{R}_{>0},\ x \mapsto \sum_{k=0}^{\infty} \frac{x^k}{k!}$ is bijective, and $\exp^{-1}$ is denoted by ln and called the *natural logarithm*.
- $f\colon \mathbb{R} \longrightarrow \mathbb{R}_{\geq 0},\ x \mapsto x^2$ is not injective, but surjective. Its restriction $f|_{\mathbb{R}_{\geq 0}}$ is bijective.

2 What Do We Need Codomains For?

The notions of surjectivity and inverse require a codomain, and we will deal with this in the next section in full detail.

Codomains are also sometimes used to express that a function does not assume certain values, for example postulating $0 \notin \operatorname{cod}(f)$ guarantees that $\frac{1}{f(x)}$ is defined for all $x \in \operatorname{dom}(f)$. Clearly, the same can be achieved by postulating $0 \notin \operatorname{img}(f)$, which is also more to the point: we want to exclude that f ever assumes 0, so we postulate that 0 is not in the set of values that f assumes.

I am not aware of any other uses of codomains.

3 Removing the Notion of Surjectivity

3.1 Proposition. A function f is injective if and only if there is $g\colon \operatorname{img}(f) \longrightarrow \operatorname{dom}(f)$ with:

$$\forall x \in \operatorname{dom}(f) :\ g(f(x)) = x \qquad (*)$$

If such a g exists, it is uniquely determined.

Proof. Define $\varphi\colon \operatorname{dom}(f) \longrightarrow \operatorname{img}(f),\ x \mapsto f(x)$; this is f where we replaced $\operatorname{cod}(f)$ by $\operatorname{img}(f)$. Clearly, φ is surjective, hence:

$$\begin{aligned}
& f \text{ is injective} \\
\iff\ & \varphi \text{ is bijective} \\
\iff\ & \exists g\colon \operatorname{cod}(\varphi) \longrightarrow \operatorname{dom}(\varphi) :\ g \circ \varphi = \mathrm{id}_{\operatorname{dom}(\varphi)} \wedge \varphi \circ g = \mathrm{id}_{\operatorname{cod}(\varphi)} \\
\iff\ & \exists g\colon \operatorname{img}(f) \longrightarrow \operatorname{dom}(f) :\ g \circ \varphi = \mathrm{id}_{\operatorname{dom}(f)} \wedge \varphi \circ g = \mathrm{id}_{\operatorname{img}(f)} \\
\iff\ & \exists g\colon \operatorname{img}(f) \longrightarrow \operatorname{dom}(f) :\ (\forall x \in \operatorname{dom}(f) :\ g(\varphi(x)) = x) \wedge (\forall y \in \operatorname{img}(f) :\ \varphi(g(y)) = y) \\
\iff\ & \exists g\colon \operatorname{img}(f) \longrightarrow \operatorname{dom}(f) :\ (\forall x \in \operatorname{dom}(f) :\ g(f(x)) = x) \wedge (\forall y \in \operatorname{img}(f) :\ f(g(y)) = y)
\end{aligned}$$

In the final statement, the second conjunct is implied by the first: given $y \in \operatorname{img}(f)$, say $y = f(x)$ with $x \in \operatorname{dom}(f)$, we obtain:

$$f(g(y)) = f(g(f(x))) = f(x) = y$$

Hence the final statement is equivalent to the existence of $g\colon \operatorname{img}(f) \longrightarrow \operatorname{dom}(f)$ with property $(*)$.

Now let $g, h\colon \operatorname{img}(f) \longrightarrow \operatorname{dom}(f)$ be functions with property $(*)$. Let $y \in \operatorname{img}(f)$, say $y = f(x)$ with $x \in \operatorname{dom}(f)$. Then:

$$g(y) = g(f(x)) = x = h(f(x)) = h(y) \qquad \square$$

If f is injective, the function g in the preceding proposition can easily take the place of the inverse in our thinking. The proof shows how easy it is to "make" an injective function bijective: keep the domain, keep the mapping rule, and reduce the codomain to the image.

In my experience, the notion of surjectivity causes a lot of trip-ups in introductory classes. After understanding injectivity, many students intuitively conceive Proposition 3.1, at least in spirit. But surjectivity throws them off; this notion appears contrived and unnecessary, and looking at Proposition 3.1, I think we should admit that surjectivity probably is contrived and unnecessary indeed. At the time of writing, for several semesters I have taught mathematics, on different levels, using codomain-free functions and the notion of inverse function given via Proposition 3.1. It works.

There is prior art for this: in [5], Section 1.5, the inverse function is defined for injective functions; injectivity is called "one-to-one" there, and the image is called the "range" there. Very similar approaches are taken in:

- [4], Chapter 12, Theorem 1
- [2], Section 4.1 (in German)

Please let me know of further references.

The following three sections demonstrate more problems with codomains, some of them being serious.

4 Problem: Restricting the Domain

Define:

$$\operatorname{Exp}\colon \mathbb{C} \longrightarrow \mathbb{C}_{\neq 0},\ z \mapsto \sum_{k=0}^{\infty} \frac{z^k}{k!} \qquad \textit{complex exponential function}$$

It would be nice to write $\operatorname{Exp}|_{\mathbb{R}} = \exp$, but this is false, since:

$$\operatorname{cod}(\operatorname{Exp}|_{\mathbb{R}}) = \mathbb{C}_{\neq 0} \qquad \operatorname{cod}(\exp) = \mathbb{R}_{>0}$$

We would need to change the codomain as well; an operation for this is already known [1]. Define:

$$f|^B\colon \operatorname{dom}(f) \longrightarrow B,\ x \mapsto f(x) \qquad \textit{corestriction} \text{ of } f \text{ onto set } B \text{ with } \operatorname{img}(f) \subseteq B \subseteq \operatorname{cod}(f)$$

Then:

$$(\operatorname{Exp}|_{\mathbb{R}})|^{\mathbb{R}_{>0}} = \exp$$

So this problem can be fixed, although at the price of a more complicated notation. The following two sections show more serious problems.

5 Problem: Vector Space of Functions

Define:

$$V := \{f \mid \operatorname{dom}(f) = \operatorname{cod}(f) = \mathbb{R}\}$$
$$f + g\colon \mathbb{R} \longrightarrow \mathbb{R},\ x \mapsto f(x) + g(x) \qquad (f, g \in V)$$
$$\lambda f\colon \mathbb{R} \longrightarrow \mathbb{R},\ x \mapsto \lambda f(x) \qquad (\lambda \in \mathbb{R},\ f \in V)$$

With these operations, V becomes a vector space over $\mathbb{R}$. It will perhaps surprise the reader to note that $\exp \notin V$. Indeed, $\operatorname{cod}(\exp) = \mathbb{R}_{>0}$, but all functions in V have codomain $\mathbb{R}$. We could have defined exp using $\mathbb{R}$ as the codomain, but then exp would not have been surjective and hence there would be no ln (unless we get rid of the notion of surjectivity as suggested in Section 3). So this is not an option.

Let us try to fix this, maintaining the notion of codomain. Define:

$$W := \{f \mid \operatorname{dom}(f) = \mathbb{R} \wedge \operatorname{cod}(f) \subseteq \mathbb{R}\}$$

Now $\exp \in W$. But the above operations do not turn W into a vector space anymore. We have, closely observing the definition of addition, $f + \exp \neq \exp$ for all $f \in W$, since $\operatorname{cod}(f + \exp) = \mathbb{R}$, hence there is no neutral element. Also, closely observing the definition of scalar multiplication, we recognize that $\operatorname{cod}(1 \cdot \exp) = \mathbb{R}$, hence $1 \cdot \exp \neq \exp$. And so on; it is a mess.

The problem appears to be that the operations impose $\mathbb{R}$ as codomain of $f + g$ and λf. Define instead:

$$f + g\colon \mathbb{R} \longrightarrow B_{f+g},\ x \mapsto f(x) + g(x) \qquad B_{f+g} := \{f(x) + g(x) \mid x \in \mathbb{R}\}$$
$$\lambda f\colon \mathbb{R} \longrightarrow B_{\lambda f},\ x \mapsto \lambda f(x) \qquad B_{\lambda f} := \{\lambda f(x) \mid x \in \mathbb{R}\}$$

This approach makes the codomain just as large as it needs to be. But there is still no neutral element. The only candidates for neutral elements are the functions $\nu_B\colon \mathbb{R} \longrightarrow B,\ x \mapsto 0$ with $0 \in B \subseteq \mathbb{R}$. But we have $\nu_{\mathbb{R}} + \nu_B = \nu_{\{0\}} \neq \nu_{\mathbb{R}}$, so none of them is neutral.

The new codomains appear too restrictive; perhaps this is the problem. Define instead:

$$f + g\colon \mathbb{R} \longrightarrow B_{f+g},\ x \mapsto f(x) + g(x) \qquad B_{f+g} := \{f(x) + g(x) \mid x \in \mathbb{R}\} \cup \operatorname{cod}(f) \cup \operatorname{cod}(g)$$
$$\lambda f\colon \mathbb{R} \longrightarrow B_{\lambda f},\ x \mapsto \lambda f(x) \qquad B_{\lambda f} := \{\lambda f(x) \mid x \in \mathbb{R}\} \cup \operatorname{cod}(f)$$

Note how complicated this has become. And it is still not a solution, because now $\exp + \nu_B \neq \exp$, for all $0 \in B \subseteq \mathbb{R}$, since $0 \in \operatorname{cod}(\exp + \nu_B)$ and $0 \notin \operatorname{cod}(\exp)$. So there is still no neutral element.

As a last attempt, define:

$$f + g\colon \mathbb{R} \longrightarrow B_{f+g},\ x \mapsto f(x) + g(x) \qquad B_{f+g} := \{f(x) + g(x) \mid x \in \mathbb{R}\} \cup (\operatorname{cod}(f) \cap \operatorname{cod}(g))$$
$$\lambda f\colon \mathbb{R} \longrightarrow B_{\lambda f},\ x \mapsto \lambda f(x) \qquad B_{\lambda f} := \{\lambda f(x) \mid x \in \mathbb{R}\} \cup \operatorname{cod}(f)$$

Finally, $\nu_{\mathbb{R}}$ is neutral. But trouble continues with additive inverse elements. The only candidates for an additive inverse to exp are the functions $\mu_B\colon \mathbb{R} \longrightarrow B,\ x \mapsto -\exp(x)$ with $]-\infty, 0[\subseteq B \subseteq \mathbb{R}$. We have $\operatorname{cod}(\exp + \mu_B) = \{0\} \cup (\mathbb{R}_{>0} \cap B) \subseteq \mathbb{R}_{\geq 0}$, hence $\exp + \mu_B \neq \nu_{\mathbb{R}}$, regardless of what B is.

I will stop here. If you have any idea how to fix this while maintaining the notion of codomain, please let me know.

6 Problem: Derivatives

Let f be a function with $\operatorname{dom}(f) = \mathbb{R}$ and $\operatorname{cod}(f) \subseteq \mathbb{R}$, and let f be differentiable, that is, there exists $f'(x) := \lim_{\hat{x} \to x} \frac{f(\hat{x}) - f(x)}{\hat{x} - x} \in \mathbb{R}$ for all $x \in \mathbb{R}$. What codomain should f' have?

- $\operatorname{cod}(f') = \mathbb{R}$? Then $\exp' \neq \exp$, which most likely is not what we want.
- $\operatorname{cod}(f') = \operatorname{cod}(f)$? This does not work for $\mathbb{R} \longrightarrow \mathbb{R}_{\geq 0},\ x \mapsto x^2$.
- $\operatorname{cod}(f') = \{f'(x) \mid x \in \mathbb{R}\}$? Then all derivatives are surjective, hence $\mathbb{R} \longrightarrow \mathbb{R},\ x \mapsto x^2$ does not admit an anti-derivative, which most likely is not what we want.
- $\operatorname{cod}(f') = \{f'(x) \mid x \in \mathbb{R}\} \cup \operatorname{cod}(f)$? Then $\mathbb{R} \longrightarrow \mathbb{R}_{\geq 0},\ x \mapsto x^2$ does not admit an anti-derivative, because any such anti-derivative would be of the form $\mathbb{R} \longrightarrow \mathbb{R},\ x \mapsto \frac{1}{3}x^3 + c$ with $c \in \mathbb{R}$, which yields codomain $\mathbb{R}$.

I will stop here. I do not think there is a solution, except for dropping the notion of codomain. If you have any idea how to fix this while maintaining the notion of codomain, please let me know.

7 Solution

Just drop the notion of codomain and adopt the notion of inverse function provided by Proposition 3.1; we keep the notation f^{-1} for this inverse. Equality of functions is defined as before, only without the codomain condition:

$$f = g :\iff \operatorname{dom}(f) = \operatorname{dom}(g) \wedge (\forall x \in \operatorname{dom}(f) : f(x) = g(x))$$

All the problems demonstrated above will go away.

Notation could be simplified to any of the following, where A is the domain:

(1) $f\colon A,\, x \mapsto E$

(2) $f\colon x \in A \mapsto E$

(3) $f\colon x \mapsto E \quad (x \in A)$

(4) $f(x) = E \quad (x \in A)$

The notation given in **(2)** is perhaps the best for inline notation of anonymous functions. For example $x \in \mathbb{R} \mapsto x^2$ is the function that maps each real to its square. As always, we can use parentheses to avoid ambiguities or for typographical reasons. For example, we could write $(x \in \mathbb{R} \mapsto x^2)$ to emphasize that this is one mathematical object. Another example is $n \in \mathbb{N} \mapsto (x \in \mathbb{R} \mapsto x^n)$, which can also be written as $(n \in \mathbb{N} \mapsto (x \in \mathbb{R} \mapsto x^n))$, and which is the function that maps each positive integer n to the function that maps each real to its nth power.

While we are at it, we can as well make notation more systematic in another aspect too. Instead of the colon after the function name, we use standard notation for definition and statement of equality. To *define* f as a certain function, write $f := (x \in A \mapsto E)$. To *state* that a *previously defined* object f equals a certain function, write $f = (x \in A \mapsto E)$.

There is prior art for this: in [3], Section 5.2, our notation **(2)** is introduced (using square brackets there).

By $(A \longrightarrow B)$ we could denote the set of all functions f with $\operatorname{dom}(f) = A$ and $\operatorname{img}(f) \subseteq B$. Note $(A \longrightarrow B) \subseteq (A \longrightarrow B')$ for all $B \subseteq B'$. This notation (with square brackets) is also used in [3].

Examples:

- $(x \in \mathbb{R}_{\geq 0} \mapsto x^2) \in (\mathbb{R}_{\geq 0} \longrightarrow \mathbb{R}_{\geq 0}) \subseteq (\mathbb{R}_{\geq 0} \longrightarrow \mathbb{R})$
- $(x \in \mathbb{R} \mapsto (x+1)^2) = (x \in \mathbb{R} \mapsto x^2 + 2x + 1)$
- $(x \in \mathbb{R}_{\geq 0} \mapsto x^2)^{-1} = (x \in \mathbb{R}_{\geq 0} \mapsto \sqrt{x})$
- $\operatorname{img}(x \in \mathbb{R}_{>0} \mapsto -\frac{1}{x}) = \left]-\infty, 0\right[$
- $(x \in \mathbb{R} \mapsto x^2)' = (x \in \mathbb{R} \mapsto 2x)$
- $(\mathbb{R} \longrightarrow \mathbb{R})$ becomes a vector space over $\mathbb{R}$ when endowed with the operations:

  $$f + g := (x \in \mathbb{R} \mapsto f(x) + g(x))$$
  $$\lambda f := (x \in \mathbb{R} \mapsto \lambda f(x))$$

  For example:

  $$(x \in \mathbb{R} \mapsto x - 1) + (x \in \mathbb{R} \mapsto x^2) = (x \in \mathbb{R} \mapsto x^2 + x - 1)$$
  $$2 \cdot (x \in \mathbb{R} \mapsto x - 1) = (x \in \mathbb{R} \mapsto 2x - 2)$$

- $(x \in \mathbb{R}_{\geq 0} \mapsto \sqrt{x}) \circ (x \in \mathbb{R} \mapsto x^2 + 1) = (x \in \mathbb{R} \mapsto \sqrt{x^2+1})$

8 Functions as Sets

It can be argued that the mapping rule is a somewhat informal concept. The following helps. A set f is called a *function* if there are sets A and B such that $f \subseteq A \times B$ and f is *right-unique*, that is:

$$\forall (a,b), (a,c) \in f : b = c$$

Domain and image are easily defined:

$$\operatorname{dom}(f) := \{x \in A \mid \exists y : (x,y) \in f\} \qquad \operatorname{img}(f) := \{y \in B \mid \exists x : (x,y) \in f\}$$

Clearly, $\operatorname{dom}(f)$ and $\operatorname{img}(f)$ are independent of the choice of A and B, as long as $f \subseteq A \times B$. It makes sense to define the *inverse* of f as $f^{-1} := \{(y,x) \mid (x,y) \in f\}$, regardless of whether f is injective or not. It is easy to prove for all functions f:

$$f^{-1} \text{ is a function } \iff f \text{ is injective}$$

Nowhere do we require a codomain.

9 If We Need Something Like Surjectivity

We could say that a function f is *surjective on* a set B if $B = \operatorname{img}(f)$, or, maybe, if $B \subseteq \operatorname{img}(f)$. For example, a permutation on a set S is a function in $(S \longrightarrow S)$ that is injective, and surjective on S. We could also say that a function f is *bijective* between a set A and a set B if $\operatorname{dom}(f) = A$ and $\operatorname{img}(f) = B$ and f is injective. Then a permutation on a set S is a function that is bijective between S and S. Or a vector space isomorphism between vector spaces V and W is a linear map that is bijective between V and W. This way, a text which frequently uses the surjectivity or bijectivity notion with respect to a codomain can be quickly adapted to the codomain-free framework.

10 Discussion

We have seen how the notion of codomain evokes serious problems (Section 5 and Section 6) and can safely be disposed of even in the context of inverse functions (Section 3). There seems to be a silent agreement in the mathematical community to ignore codomains most of the time, and to consider two functions equal if and only if they have the same domain and assume the same value in every argument. Only when it comes to surjectivity and inverse functions is attention paid to codomains. This may work well for most purposes if done by seasoned mathematicians. But it understandably causes confusion in introductory classes, and it is unnecessarily inconsistent. Even the notion of surjectivity, although not having any contradictions in itself, is often perceived as contrived and superfluous by students. Indeed, a concept of inverse function can be introduced just fine using only domain, mapping rule, image, and injectivity (Proposition 3.1).

When considering functions as sets in the codomain-free framework, matters become particularly elegant. A function then is simply a right-unique subset of some Cartesian product; all other parameters, such as domain and image, can be deduced from this set.

Conclusion: I suggest that we drop the notion of codomain; a function should simply consist of a domain and a mapping rule. Prior art for this is given in [2–5], which includes references from calculus and computer science. As notation, I suggest using the one from [3], but without the square brackets, that is, $x \in A \mapsto E$ or $(x \in A \mapsto E)$, where A is the domain. A function f should have an inverse as soon as f is injective; the inverse always has $\operatorname{img}(f)$ as its domain.

References

[1] *Corestriction*. Wikipedia, The Free Encyclopedia. 2024. URL: https://en.wikipedia.org/w/index.php?title=Corestriction&oldid=1246524920 (visited on 2026-07-31).

[2] K. Königsberger. *Analysis 1*. German. 6th ed. 2004.

[3] L. Lamport. *Specifying Systems: The TLA+ Language and Tools for Hardware and Software Engineers*. 2003. URL: https://lamport.azurewebsites.net/tla/book-21-07-04.pdf.

[4] M. Spivak. *Calculus*. 4th ed. 2008.

[5] J. Stewart. *Calculus: Early Transcendentals*. 8th ed. 2016.